Fractions

Multiplication of Fractions

We know multiplication is nothing but repetitive addition, it implies to both whole numbers and fractions.
In case of whole numbers, 4 x 5 = 4 + 4 + 4 + 4 + 4 = 20
In case of fractions

$$3 X \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1+1+1}{5} = \frac{3}{5}$$

Simple way to multiply fractions is multiply numerator with the whole number and leave denominator as it is.

Example 1. Multiply 5 x ⅗.

Solution.

$$5 x \frac{3}{5} = \frac{5 X 3}{5} = \frac{15}{5} = 3$$

Example 2. Multiply 2 x ⅝.

Solution.

$$2 X \frac{5}{7} = \frac{2 X 5}{7} = \frac{10}{7}$$

Multiplication of Mixed Fraction With Whole Number

Multiplication of mixed fraction with whole number can be done by two ways.

1. Repetitive addition
2. Converting mixed fraction into improper fraction

Repetitive addition

Example 1. Multiply $3\frac{3}{5}$ by 5.

Solution.

$$3\frac{3}{5}+3\frac{3}{5}+3\frac{3}{5}+3\frac{3}{5}+3\frac{3}{5}$$

$$=3+3+3+3+3+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}$$

$$=15+\frac{3+3+3+3+3}{5}=15+\frac{15}{5}=15+3=18$$

Converting mixed fraction into improper fraction

Example 1. Multiply $2\frac{3}{5}$ X 4

Solution. First convert the mixed fraction into improper fraction

$$2\frac{3}{5}=\frac{5x2+3}{5}=\frac{13}{5}$$

Then multiply $\frac{13}{5}$ by 4

$$\frac{13}{5}X4=\frac{13x4}{5}=\frac{52}{5}$$

Multiplication of Mixed Fraction With Fraction

In this type of multiplication, we should convert the mixed fraction into improper fraction. Then, we must multiply numerators of both the fractions and keep the result in numerator of the result. After that, denominators should be multiplied, and result should be kept in denominator of the result.

Example 1. Multiply $2\frac{3}{5}$ by $\frac{3}{4}$.

Solution.

$$2\frac{3}{5}X\frac{3}{4}=\frac{5x2+3}{5}X\frac{3}{4}=\frac{13}{5}x\frac{3}{4}=\frac{13x3}{5x4}=\frac{39}{20}$$

So, the answer is $\frac{39}{20}$.

Example 2. Multiply $3\frac{1}{5}$ by $\frac{5}{8}$.

Solution.

$$3\frac{1}{5} \times \frac{5}{8} = \frac{5x3+1}{5} \times \frac{5}{8} = \frac{16}{5} \times \frac{5}{8} = \frac{16 \times 5}{5 \times 8} = \frac{80}{40} = 2$$

So, the answer is 2.

Multiplication of Mixed Fractions

When we multiply two mixed fractions, then we must convert both the mixed fractions into improper fraction and then multiply numerator with numerator and denominator with denominator of the improper fraction.

Example 1. Multiply $2\frac{1}{5}$ by $1\frac{2}{11}$.

Solution. Convert the mixed fractions to improper fraction.

$$2\frac{1}{5} \times 1\frac{2}{11} = \frac{\cancel{11}}{5} \times \frac{13}{\cancel{11}}$$

11 is present in both numerator and denominator, we can cancel them out as shown above

$$= \frac{13}{5} = 2\frac{3}{5}$$

Thus, the answer is $2\frac{3}{5}$.

Fraction of a Whole Number

Fraction of a whole number is similar to the product of fraction with a whole number. Let's have a look at some examples.

Example 1. Number of students in a class is 60. If girl's strength is $\frac{1}{3}$ of the total number of students, then what is the number of girl students?

Solution. Number of girls present in the class

$$= 60 \times \frac{1}{3} = \frac{60}{3} = 20$$

So, total number of girl student is 20.

Example 2. Population of a city is 25545. If $\frac{1}{5}$ th of the population are senior citizens, then what is the number of senior citizens leaving in that city?

Solution. Number of senior citizens leaving in the city

$$= 25545 \times \frac{1}{5} = \frac{25545}{5} = 5109$$

Product of More Fractions

In this type of multiplication, we multiply more than two fractions. Let's have a look at some examples.

Example 1. Find the product of $\frac{3}{5}$, $\frac{7}{3}$ and $\frac{2}{7}$.

Solution. First find the product of any two fractions.

$$\frac{\cancel{3}}{5} \times \frac{7}{\cancel{3}} = \frac{7}{5}$$

Then multiply remaining fraction with $\frac{7}{5}$

$$= \frac{\cancel{7}}{5} \times \frac{2}{\cancel{7}} = \frac{2}{5}$$

Thus, the product of $\frac{3}{5}$, $\frac{7}{3}$ and $\frac{2}{7}$ is equal to $\frac{2}{5}$.

Example 2. Find the product of $\frac{4}{5}$, $\frac{7}{3}$, $\frac{5}{7}$ and $2\frac{5}{15}$.

Solution. First convert mixed fraction into improper fraction and then do the multiplication.

$$\frac{4}{\cancel{5}} \times \frac{\cancel{7}}{3} \times \frac{\cancel{5}}{\cancel{7}} \times \frac{35}{15} = \frac{4 \times \cancel{35}\,7}{3 \times \cancel{15}\,3} = \frac{28}{9} = 3\frac{1}{9}$$

Thus, the result is $3\frac{1}{9}$.

Properties of Fraction Multiplication

1. The product of a fraction and zero is zero.

2. The product of a fraction and 1 is the fraction itself.

3. Changing the order of multiplication of the fractions does not change the result.

4. Changing the grouping in the multiplication of three or more fractions does not change the result.

Reciprocal of Fraction

If the numerator and denominator of a fraction are interchanged, we get the reciprocal of the fraction.

Example 1. Find out the reciprocal of $\frac{3}{5}$.

Solution. Here numerator is 3 and denominator 5. By interchanging the numerator and the denominator, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

Division of Fractions

Division of fraction can be performed by following below mentioned steps.

1. Find the reciprocal of the divisor by interchanging the numerator and denominator.

2. Multiply the dividend with the reciprocal of the divisor.

Example 1. Divide $\frac{3}{5}$ by $\frac{1}{5}$.

Solution. Reciprocal of $\frac{1}{5}$ is 5.

$$\frac{3}{5} \div \frac{1}{5} = \frac{3}{5} \times 5 = 3$$

Thus, the result is 3.

Example 2. Divide $\frac{3}{11}$ by 11.

Solution. Reciprocal of 11 is $\frac{1}{11}$.

$$\frac{3}{11} \div 11 = \frac{3}{11} \times \frac{1}{11} = \frac{3}{121}$$

Thus, the answer is $^{3}/_{121}$.

Example 3. Divide $5^{1}/_{2}$ by $2^{1}/_{3}$.

Solution. Convert the mixed fractions into improper fractions.

$$5\frac{1}{2} = \frac{11}{2} \qquad 2\frac{1}{3} = \frac{7}{3}$$

$$5\frac{1}{2} \div 2\frac{1}{3} = \frac{11}{2} \div \frac{7}{3} \text{ (Reciprocal of } \frac{7}{3} \text{ is } \frac{3}{7}\text{)}$$

$$= \frac{11}{2} \times \frac{3}{7} = \frac{11x3}{2x7} = \frac{33}{14} = 2\frac{5}{14}$$

Thus, the answer is $2^{5}/_{14}$.

Division of Whole Number by Fraction

To divide a whole number by a fraction, we must multiply the reciprocal of the fraction with the whole number. Let's have a look at some examples.

Example 1. Divide 8 by $^{2}/_{3}$.

Solution. Reciprocal of $^{2}/_{3}$ is $^{3}/_{2}$.

$$8 \div \frac{2}{3} = \cancel{8}^{4} \times \frac{3}{\cancel{2}} = 4 \times 3 = 12$$

Thus, the answer is 12.

Example 2. Divide 9 by $1^{1}/_{2}$.

Solution. Convert the mixed fraction into improper fraction = $1^{1}/_{2} = {}^{3}/_{2}$

$$\cancel{9}^{3} \times \frac{2}{\cancel{3}} = 3 \times 2 = 6$$

Reciprocal of $^{3}/_{2}$ is $^{2}/_{3}$.
Thus, the answer is 6.

Properties of Fraction Division

1. When zero is divided by a fraction, then the result will be zero.

2. When fraction is divided by 1, then the result will be the fraction itself.

3. When 1 is divided by a fraction, then the result will be the reciprocal of the fraction.

4. When a fraction is divided by itself, then the result will be one.